Written Exam
(2 hours)

General remarks:

- Open book exam.
- Switch off your mobile phone!
- Do not stay too long on a part where you experience a lot of difficulties.
- Use only the separate sheet for your answers.
- If not stated otherwise, all tests have to be done at the 5%-level.
- The tables of critical values for the F and t distributions are attached at the end.
- Exercises 4 and 5 are multiple-choice exercises. In each sub-exercise, exactly one answer is correct. A correct answer adds 1 *plus*-point and a wrong answer $\frac{1}{2}$ *minus*-point. You get a minimum of 0 points for each multiple-choice exercise. Tick the correct answer to the multiple choice exercises in the separately added answer sheet.

Good Luck!

1. (12 Points)

Four shops, Migros, Globus, Manor and Coop, were selected by Marius, an MTEC student, for a market research study of their Christmas advertising success. Four typical customers were randomly selected at each store on Saturday, December 20th, for a total of 16 people. The advertising success was defined as the number of items purchased in the given shop by the customer. The following table displays the collected data:

Migros	Coop	Globus	Manor
3	7	5	5
0	2	0	10
1	3	4	8
4	6	2	8

We use the following model:

$$Y_{ij} = \mu + A_i + \epsilon_{ij}$$

where $i = Migros, Globus, Manor, Coop$ and $j = 1, \ldots, 4$. We assume the standard assumptions about the errors ϵ_{ij}. Additionally, we choose $Coop$ as a reference level and we set $A_{Coop} = 0$.

a) Compute the estimates $\hat{\mu}$ and all effects $\hat{A}_i$.

b) Compute the sum of the residuals $\sum_{j=1}^{4} r_{ij}$ for the shop Manor.

c) Marius wants to analyse his data and decides to do some t-tests for all possible pairs of means between shops. How many such tests would that be? Explain to Marius why this is not a good idea.

d) Convinced by your argument, Marius hires you to do the analysis for him. Compute the MS_A.

e) Does the success of their Christmas advertising campains differ significantly between the four shops? Motivate your answer by clearly giving the null hypothesis, the test statistic and the critical value.

f) Marius notices that there might be a lot of variability between individuals and would like to take that into account for a new experiment. What better design would you propose him to use?

2. (13 Points) A pharmaceutical company wants to study the effect of alcohol consumption in conjunction with two types of drugs, A and B. They consider the following *treatments*:

1. Control
2. Drug A alone
3. Drug A and alcohol consumption 1 hour before
4. Drug A and alcohol consumption 1 hour after
5. Drug B alone
6. Drug B and alcohol consumption 1 hour before
7. Drug B and alcohol consumption 1 hour after

A balanced randomized block design was used, with the blocking factor being the town in which the experiment was done. Every individual in the experiment was given to test one treatment only and the effect of the drug was measured on some scale and recorded as variable Y.

a) We want to ask precise questions about the data and use contrasts to do so. Propose some contrasts to test the following things:

- $L1$: The difference between the drug A and the drug B
- $L2$: The effect of alcohol
- $L3$: The difference between taking alcohol before and after

b) Are the previous 3 contrasts orthogonal to each other? Justify your answer.

c) In general, what is the advantage of using orthogonal contrasts?

d) We fit the model and obtain the following incomplete R-Output. Is the contrast `L3` significant? Motivate your answer. (For this task the critical value of the F-statistics is 3.85).

```
                 Df  Sum Sq Mean Sq F value  Pr(>F)
Treatment         6 4738965  789828  248.62 < 2e-16
  Treatment: L1      1382201
  Treatment: L2       418775
  Treatment: L3        35885
Town             11  147560   13415    4.22 4.1e-06
Residuals       822 2611384    3177
```

e) How many people were tested in each town?

f) What do the two following contrasts test? Explain with words.

- $L4$: (6, -1, -1, -1, -1, -1, -1)
- $L5$: (0, 0, +1, -1, 0, -1, +1)

g) Calculate the F-statistics of the treatment if `Town` was taken out of the model.

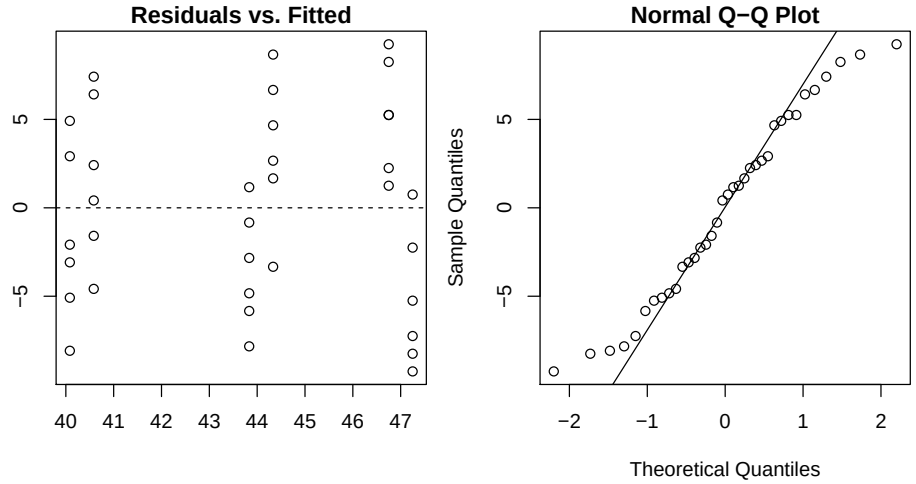
h) The factor `Town` can be modelled as a fixed or random effect. In what case would it make sense to say it is a fixed effect?

3. (12 Points) A pizzeria wants to optimize its least sold Pizza Magherita to guarantee a maximum taste experience. To find the best combination of baking temperature and baking time, they perform an experiment with two factors (temp = 180 °C, 210 °C and 240 °C, time = 10 min and 15 min). Each pizza gets judged 6 times on a scale of 1 to 100 where 100 corresponds to maximum taste experience. As analysis a two-way anova is performed and gives the following results:

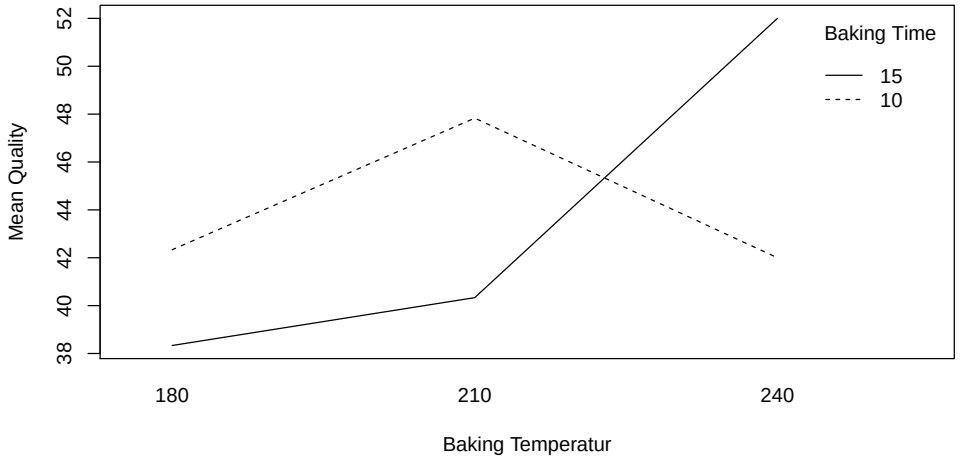
	Df	Sum Sq	Mean Sq	F value	Pr(>F)
temp	2	268	134.0	4.25	0.023
time	1	2	2.2	0.07	0.791
Residuals	32	1009	31.5		

a) From the R-Output above, what is the estimated error variance?

b) What are the standard assumptions about the errors? Are these assumptions fullfilled? Motivate your answer with help of the following residual plots.



c) Have a look at the following plot. What does the plot show? Explain why the model above can lead to wrong conclusion?



d) What are the degrees of freedom for an interaction term between baking time and baking temperature?

e) Have a look at the following output. Is the interaction effect significant? (Complete the output, if it is necessary for your answer.)

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
temp	2	268	134.0	8.13	0.0015
time	1	2	2.2	0.14	0.7145
temp:time		514			
Residuals	30	495	16.5		

f) From the output above, which is the final model you would choose and why?

g) Let's assume you want to repeat the anova after a year. However, some data got lost and the number of judgements for each pizza is not the same. Does the loss of data have an influence on the calculation of the anova? Would you recommend to use the R-function `aov`? Motivate both answers.

h) The amount of salt is known to have a quadratic relationship with the quality of the pizza dough. Explain why it could be useful to add the square root of the amount of salt as an additional variable in the anova and describe the resulting model by using a formula.

4. (7 Points) Some experts think that for the production of bread the following variables are most influencing:

	Factors	(-)	(+)
A	Mixer	Mixer1	Mixer2
B	Salt	20 g	40 g
C	Kneading Time	5 min	10 min
D	Dough resting time	1 h	3 h
E	Baking temperature	175 °C	200 °C

In order to find the best bread, the experts decide to use a 2^{k-l} fractional factorial design. They choose $l = 2$ and use the confounding relations

$$D = AB \quad \text{and} \quad E = ACD.$$

The best bread is the one that gets the most points from experts.

1) How many different breads need to be baked?
   a) 64
   b) 32
   c) 24
   d) 10
   e) 8
   f) None of the above.
2) The experts start the test with the combination Mixer2, 40 g salt, kneading 10 min, 3 h rest and baking at 200 °C (+++++). Which of the following combination is **not** tested?
   a) Mixer2, 20 g salt, kneading 5 min, 1 h rest and baking at 200 °C (ABCDE = (+ - - - +))
   b) Mixer1, 40 g salt, kneading 10 min, 1 h rest and baking at 200 °C (ABCDE = (- + + - +))
   c) Mixer2, 40 g salt, kneading 5 min, 3 h rest and baking at 175 °C (ABCDE = (+ + - + -))
   d) Mixer1, 20 g salt, kneading 5 min, 1 h rest and baking at 200 °C (ABCDE = (- - - - +))
   e) All combinations above are tested.
   f) The given information in this exercise are not sufficient to answer this question.
3) Which of the following statement is true?
   a) Estimating the interaction effect ED is equal to estimating the interaction $ACBD$.
   b) Estimating the main effect B is equal to estimating the interaction DE.
   c) Estimating the interaction effect BC is equal to estimating the interaction ABD.
   d) Estimating the main effect E is equal to estimating the interaction CB.

4) What is the resolution of the experiment?

   a) 2
   b) 3
   c) 4
   d) 5

The experts provide you the following table with estimates which contains all significant effects.

$\widehat{A}$	$\widehat{B}$	$\widehat{C}$	$\widehat{D}$	$\widehat{E}$	$\widehat{AD}$
0.5	-5.2	2	3	3.8	-4.5

5) Which is the most important main effect?

   a) Choice of mixer (A)
   b) Amount of salt (B)
   c) Kneading time (C)
   d) Dough resting time (D)
   e) Baking temperature (E)

6) How need A and D to be chosen for the best bread?

   a) They need to have the both positive sign: A (-) and D (-).
   b) They need to have the both negative sign: A (+) and D (+).
   c) They need to have the opposite sign: A (+) and D (-).
   d) They need to have the opposite sign: A (-) and D (+).
   e) They can be chosen arbitrary.

7) Which combination leads to the best bread?

   a) Mixer2, 20 g salt, kneading 10 min, 3 h rest and baking at 200 °C (ABCDE = (+ - + + +))
   b) Mixer1, 40 g salt, kneading 10 min, 1 h rest and baking at 175 °C (ABCDE = (- + + - -))
   c) Mixer2, 40 g salt, kneading 5 min, 1 h rest and baking at 175 °C (ABCDE = (+ + - - -))
   d) Mixer2, 20 g salt, kneading 5 min, 3 h rest and baking at 200 °C (ABCDE = (+ - - + +))
   e) Mixer1, 20 g salt, kneading 10 min, 3 h rest and baking at 200 °C (ABCDE = (- - + + +))

5. (7 points)

1) You are given the following experimental design with blocking factors X and Y each having 4 levels and a treatment A1, A2, A3, A4. Which type of design is this?

		Blocks X			
		1	2	3	4
Blocks	1	A3	A2	A4	A1
Y	2	A1	A3	A2	A4
	3	A4	A1	A3	A2
	4	A2	A4	A1	A3

   a) Split plot design
   b) 2^3 design
   c) Latin square design
   d) Complete random design
   e) Cross-over design
   f) None of the previous designs

2) A pharmaceutical company has recently developed a new version of a medication that is already on the market. The company decides to compare the new medication with the medication currently in use by testing it on 50 patients in a medical trial. Order the following designs according to the expected carry-over effect, from the one where you expect the strongest carry-over effect to the design where you expect the weakest carry-over effect.

   (i) Each patient takes the new medication for the first half of the trial, then the old medication for the second half of the trial.
   (ii) Each patient is assigned a random medication at the beginning of the trial, and continues taking that medication until the end of the trial.
   (iii) Each patient takes the new medication on odd days of the trial and the old medication on even days of the trial.

   a) (i), (ii), (iii)
   b) (i), (iii), (ii)
   c) (ii), (i), (iii)
   d) (ii), (iii), (i)
   e) (iii), (ii), (i)
   f) (iii), (i), (ii)

3) One benefit of a cross-over design in comparison to a parallel design is that:

   a) It lowers variability.
   b) It introduces carry-over effect.
   c) Drop-out is less likely.
   d) None of the above are benefits of the cross-over design as compared to the parallel design.

4) A toothpaste company is testing 10 new toothpaste types. 15 participants have been selected and each toothpaste type needs to be tested 6 times. Which is the necessary condition for a BIBD (Balanced incomplete block design)?

   a) Each block should be of size 3.
   b) Each block should be of size 4.
   c) Each block should be of size 5.
   d) Each block should be of size 6.

5) Choose the correct answer. A BIBD is called symmetric if:

   a) The number of treatments is equal to block size.
   b) The block size is equal to the number of times each treatment occurs.
   c) The number of times any two treatments occur together in a block is equal to the number of blocks.
   d) The number of treatments is equal to the number of blocks.
   e) None of the above are true.

6) The toothpaste factory has selected 4 out of 10 types from the previous test. They are now considering these 4 types of toothpaste and 3 types of packaging. 60 participants have been selected for the experiment, and each participant is supposed to test and rate every packaging of exactly one toothpaste type on a 1-5 scale. Which type of design is this?

   a) Cross-over design
   b) Split plot design
   c) Complete random design
   d) Factorial design
   e) Latin square design
   f) None of the previous designs

7) The toothpaste company would like to minimize the costs of the experiment in the previous question by minimizing the number of measurements needed. Which type of design would require the smallest number of measurements?

   a) Fractional factorial design
   b) Complete random design
   c) Latin square design
   d) Split plot design

Table of critical values at the 5% level of the F-distributions with ν_1 degrees of freedom in the numerator and ν_2 degrees of freedom in the denominator.

ν_2 \ ν_1 =	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	161.45	199.50	215.71	224.58	230.16	233.99	236.77	238.88	240.54	241.88	242.98	243.91	244.69	245.36	245.95
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.40	19.41	19.42	19.42	19.43
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.76	8.74	8.73	8.71	8.70
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.94	5.91	5.89	5.87	5.86
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.70	4.68	4.66	4.64	4.62
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.03	4.00	3.98	3.96	3.94
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.60	3.57	3.55	3.53	3.51
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.31	3.28	3.26	3.24	3.22
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.10	3.07	3.05	3.03	3.01
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.94	2.91	2.89	2.86	2.85
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.82	2.79	2.76	2.74	2.72
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.72	2.69	2.66	2.64	2.62
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.63	2.60	2.58	2.55	2.53
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.57	2.53	2.51	2.48	2.46
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.51	2.48	2.45	2.42	2.40
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.46	2.42	2.40	2.37	2.35
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.41	2.38	2.35	2.33	2.31
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.37	2.34	2.31	2.29	2.27
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.34	2.31	2.28	2.26	2.23
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.31	2.28	2.25	2.22	2.20
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.28	2.25	2.22	2.20	2.18
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.26	2.23	2.20	2.17	2.15
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.24	2.20	2.18	2.15	2.13
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.22	2.18	2.15	2.13	2.11
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.20	2.16	2.14	2.11	2.09
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.18	2.15	2.12	2.09	2.07
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	2.17	2.13	2.10	2.08	2.06
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.15	2.12	2.09	2.06	2.04
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18	2.14	2.10	2.08	2.05	2.03
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.13	2.09	2.06	2.04	2.01
31	4.16	3.30	2.91	2.68	2.52	2.41	2.32	2.25	2.20	2.15	2.11	2.08	2.05	2.03	2.00
32	4.15	3.29	2.90	2.67	2.51	2.40	2.31	2.24	2.19	2.14	2.10	2.07	2.04	2.01	1.99
33	4.14	3.28	2.89	2.66	2.50	2.39	2.30	2.23	2.18	2.13	2.09	2.06	2.03	2.00	1.98
34	4.13	3.28	2.88	2.65	2.49	2.38	2.29	2.23	2.17	2.12	2.08	2.05	2.02	1.99	1.97
35	4.12	3.27	2.87	2.64	2.49	2.37	2.29	2.22	2.16	2.11	2.07	2.04	2.01	1.99	1.96
36	4.11	3.26	2.87	2.63	2.48	2.36	2.28	2.21	2.15	2.11	2.07	2.03	2.00	1.98	1.95
37	4.11	3.25	2.86	2.63	2.47	2.36	2.27	2.20	2.14	2.10	2.06	2.02	2.00	1.97	1.95
38	4.10	3.24	2.85	2.62	2.46	2.35	2.26	2.19	2.14	2.09	2.05	2.02	1.99	1.96	1.94
39	4.09	3.24	2.85	2.61	2.46	2.34	2.26	2.19	2.13	2.08	2.04	2.01	1.98	1.95	1.93
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.04	2.00	1.97	1.95	1.92

df	$t_{0.60}$	$t_{0.70}$	$t_{0.80}$	$t_{0.90}$	$t_{0.95}$	$t_{0.975}$	$t_{0.99}$	$t_{0.995}$
1	0.325	0.727	1.376	3.078	6.314	12.706	31.821	63.657
2	0.289	0.617	1.061	1.886	2.920	4.303	6.965	9.925
3	0.277	0.584	0.978	1.638	2.353	3.182	4.541	5.841
4	0.271	0.569	0.941	1.533	2.132	2.776	3.747	4.604
5	0.267	0.559	0.920	1.476	2.015	2.571	3.365	4.032
6	0.265	0.553	0.906	1.440	1.943	2.447	3.143	3.707
7	0.263	0.549	0.896	1.415	1.895	2.365	2.998	3.499
8	0.262	0.546	0.889	1.397	1.860	2.306	2.896	3.355
9	0.261	0.543	0.883	1.383	1.833	2.262	2.821	3.250
10	0.260	0.542	0.879	1.372	1.812	2.228	2.764	3.169
11	0.260	0.540	0.876	1.363	1.796	2.201	2.718	3.106
12	0.259	0.539	0.873	1.356	1.782	2.179	2.681	3.055
13	0.259	0.538	0.870	1.350	1.771	2.160	2.650	3.012
14	0.258	0.537	0.868	1.345	1.761	2.145	2.624	2.977
15	0.258	0.536	0.866	1.341	1.753	2.131	2.602	2.947
16	0.258	0.535	0.865	1.337	1.746	2.120	2.583	2.921
17	0.257	0.534	0.863	1.333	1.740	2.110	2.567	2.898
18	0.257	0.534	0.862	1.330	1.734	2.101	2.552	2.878
19	0.257	0.533	0.861	1.328	1.729	2.093	2.539	2.861
20	0.257	0.533	0.860	1.325	1.725	2.086	2.528	2.845
21	0.257	0.532	0.859	1.323	1.721	2.080	2.518	2.831
22	0.256	0.532	0.858	1.321	1.717	2.074	2.508	2.819
23	0.256	0.532	0.858	1.319	1.714	2.069	2.500	2.807
24	0.256	0.531	0.857	1.318	1.711	2.064	2.492	2.797
25	0.256	0.531	0.856	1.316	1.708	2.060	2.485	2.787
26	0.256	0.531	0.856	1.315	1.706	2.056	2.479	2.779
27	0.256	0.531	0.855	1.314	1.703	2.052	2.473	2.771
28	0.256	0.530	0.855	1.313	1.701	2.048	2.467	2.763
29	0.256	0.530	0.854	1.311	1.699	2.045	2.462	2.756
30	0.256	0.530	0.854	1.310	1.697	2.042	2.457	2.750
31	0.255	0.530	0.853	1.309	1.696	2.040	2.452	2.744
32	0.255	0.530	0.853	1.309	1.694	2.037	2.449	2.738
33	0.255	0.530	0.853	1.308	1.693	2.035	2.445	2.733
34	0.255	0.529	0.852	1.307	1.691	2.032	2.441	2.728
35	0.255	0.529	0.852	1.306	1.690	2.030	2.438	2.724
40	0.255	0.529	0.851	1.303	1.684	2.021	2.423	2.704
60	0.254	0.527	0.848	1.296	1.671	2.000	2.390	2.660
90	0.254	0.526	0.846	1.291	1.662	1.987	2.368	2.632
120	0.254	0.526	0.845	1.289	1.658	1.980	2.358	2.617
∞	0.253	0.524	0.842	1.282	1.645	1.960	2.326	2.576