Power

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Different Error Rates and Power

- Remember the different error rates of a statistical test:
  - **Type I** error: **Reject** $\boldsymbol{H_0}$ even though it is **true**.
  - **Type II** error: **Fail** to reject H_0 even though $\boldsymbol{H_A}$ **holds**.
- The probability of a type I error is **controlled** by the **significance level** α.
- Type II error: The probability of a type II error is typically denoted by β (it is not being controlled).
- We need to **assume** a certain setting of the parameters under H_A to be able to calculate β.
- **Power** of a statistical test is defined as

$$\text{P}[\textbf{reject } H_0 \text{ given that } H_A \text{ holds}] = 1 - \beta.$$

Power

- We are interested in power when **planning** an experiment.
- Why? We believe that H_0 is **not** true and we want to get a “significant result” out of our experimental data.
- Power tells us the probability to get a significant result if H_A is actually true.
- Calculating power is like a “thought experiment”, we do **not** need data, but a **precise specification** of the parameter setting under H_A that we believe in .

Power

- Very low power means: chances are high that you will **not** get a significant result, even though H_A holds (that is H_0 is **not** true).
- In other words: you **wasted time and money** with your experiment as it was **a priori clear** that (with high probability) the result will **not** be significant.
- If power is 0.2 it means that only in 1 out of 5 cases the result will be significant.
- Ideally, power should be of the order of $\geq 80\%$.

Power

- Power depends on
  - design of the experiment (balanced, unbalanced, …)
  - significance level α (typically $0.01, 0.05$)
  - parameter setting under the alternative (incl. error variance σ^2)
  - sample size n
- General rule: the more observations you have, the larger power.
- Typically we choose n such that power is at the appropriate level.
- If n is smaller, the experiment does **not** satisfy our needs, if n is larger, it is a waste of resources.

Power: Calculations

- For easy situations (like the two-sample t-test, one-way ANOVA, …) formulas can be derived using new terminology like a so called non-central F-distribution.
- There are some special functions, like `power.t.test` (comes with R) or the package `pwr`.
- For more complex designs, **simulations** have to be used.
- This used to be a problem in the old days but is easy nowadays.

Example: One-Way ANOVA (Roth, 2013)

- Say we want to compare $g = 5$ different treatments labelled A, B, C, D, E.
- Previous experience tells us that $\sigma^2 = 7.5$ is a realistic value of the error variance.

- We assume that at least two groups differ by 6 units.

- If we use the model

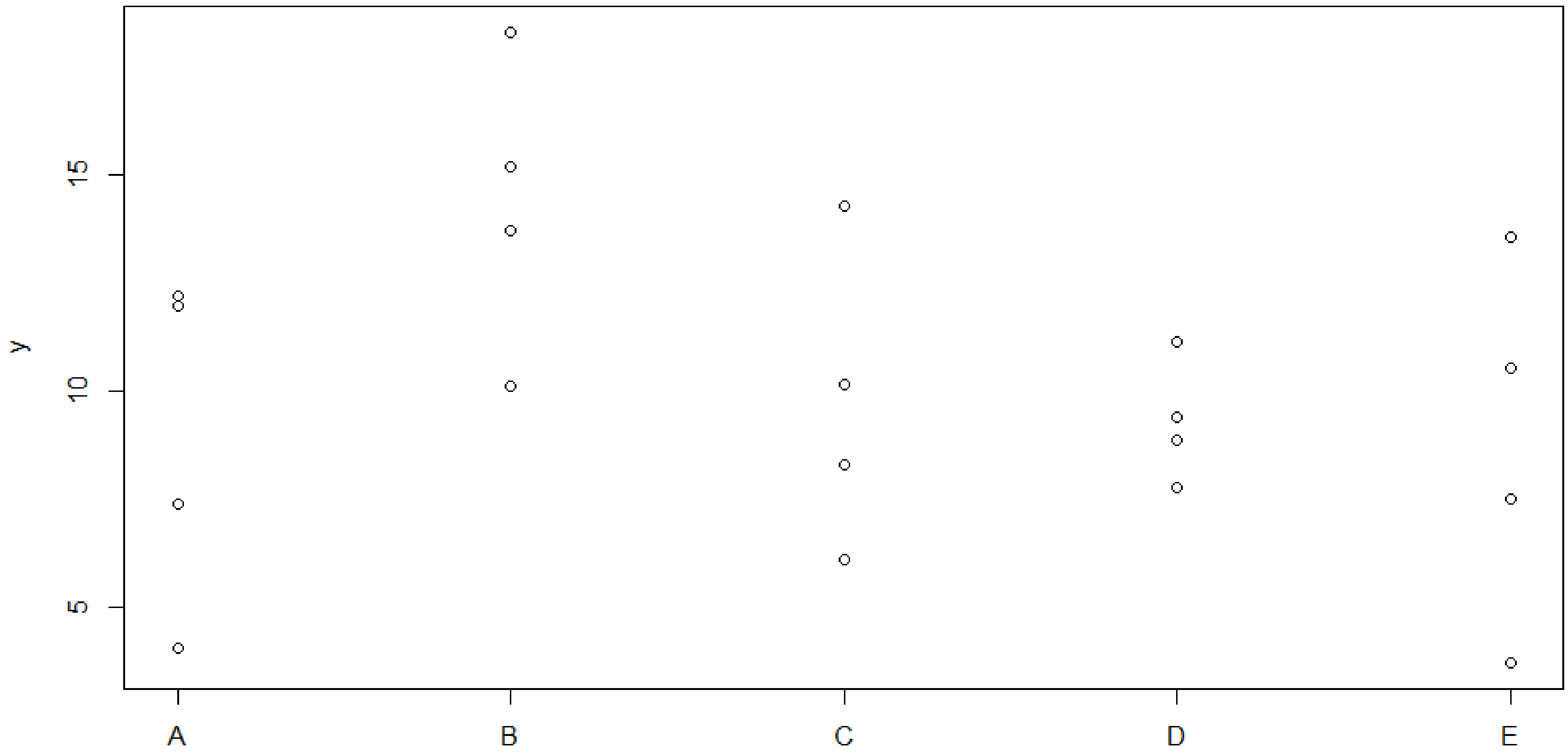
$$Y_{ij} = \mu + \alpha_i + \epsilon_{ij}$$

with the sum-to-zero constraint it means that we (e.g.) have

$$\alpha_1 = -3, \alpha_2 = 3, \alpha_3 = 0, \alpha_4 = 0, \alpha_5 = 0.$$

- See R-File for simulation.

Visualization of a Simulated Data-Set



Power Analysis: Comments

- Unfortunately, sample size is often determined by your resources (money).

- Power analysis then gives you an answer whether it is actually worth doing the study or whether it is just a waste of time.

- The difficult part is defining the parameters under the alternative, especially the error variance.

- The nice side-effect of doing a power analysis is that you actually fit your model to (simulated) data and immediately see whether the analysis “works”.